GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES

SOLUTION OF SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS USING SHEHU TRANSFORM

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ABSTRACT

Differential equations are fundamental and importance in engineering and mathematics because any physical laws and relations appear mathematically in the form of such equations. In this paper, we will discuss system of ordinary differential equations by using Shehu transform.

Keywords: *System of differential equations, Mathematics, Shehu transform, Engineering*.

I. INTRODUCTION

Many problems in engineering and science can be formulated in terms of differential equations. The ordinary differential equations arise in many areas of Mathematics, as well as in Sciences and Engineering.
Given the set of numbers a_{ij} and functions $f_i(t)$, the expression.

$$\frac{dy_i}{dt} = \sum_{j=1}^{n} a_{ij}\, y_j f_i(t) \quad (i = 1, 2, \dots, n)$$

Which is equivalent to:

$$\frac{dy_1}{dt} = a_{11}y_1 + a_{12}y_2 + \cdots + a_{1n}y_n + f_1(t)$$
$$\frac{dy_2}{dt} = a_{21}y_1 + a_{22}y_2 + \cdots + a_{2n}y_n + f_2(t)$$
$$\dots$$
$$\frac{dy_2}{dt} = a_{n1}y_1 + a_{n2}y_2 + \cdots + a_{nn}y_n + f_n(t)$$

Is called a system of linear nonhomogeneous DE with constant coefficients
In order to solve the certain system of ordinary differential equations integral transforms are widely used. In this paper, we will be discussed about the solution of system of ordinary differential equations using Shehu transform.

II. SHEHU TRANSFORM

Definition: A new transform called the Shehu transform of the function $v(t)$ belonging to a class A, where

$$A = \left\{ v(t) \colon \exists\, N, \eta_1, \eta_2 > 0, |v(t)| < N e^{\frac{|t|}{\eta_i}}, if\ t \in (-1)^i \ \times [0\ \infty) \right\}$$

Where $v(t)$ defined by $\mathbb{S}[v(t)]$ and is given by:

$$\mathbb{S}[v(t)] = V(s,u) = \int_0^\infty e^{\left(\frac{-st}{u}\right)} v(t)dt \qquad (1.1)$$

And the inverse Shehu transform is defined as

$$\mathbb{S}^{-1}[V(s,u)] = v(t) \text{for } t \geq 0 \qquad 1.2$$

DOI: https://doi.org/10.29121/gjesr.v7.i8.2020.2 Impact Factor- 5.070

Property of the SHEHU TRANSFORM

Property 1. Linearity property of Shehu transform. Let the functions $\alpha v(t)$ and $\beta w(t)$ be in set A, then $(\alpha v(t) + \beta w(t)) \in A$, where α and β are nonzero arbitrary constants, and $\mathbb{S}[\alpha v(t) + \beta w(t)] = \alpha\mathbb{S}[v(t)] + \beta\mathbb{S}[w(t)]$

Proof: Using the Definition (1.1) of Shehu transform, we get

$$\mathbb{S}[\alpha v(t) + \beta w(t)] = \int_0^\infty e^{\left(\frac{-st}{u}\right)}(\alpha v(t) + \beta w(t))dt \qquad 1.3$$

$$= \int_0^\infty e^{\left(\frac{-st}{u}\right)}\alpha v(t)dt \int_0^\infty e^{\left(\frac{-st}{u}\right)}\beta w(t)dt$$

$$= \alpha\int_0^\infty e^{\left(\frac{-st}{u}\right)}v(t)dt + \beta\int_0^\infty e^{\left(\frac{-st}{u}\right)}w(t)dt$$

$$= \alpha\mathbb{S}[v(t)] + \beta\mathbb{S}[w(t)]$$

Property 2. Let the function $v(\beta t)$ be in set A, where β is an arbitrary constant. Then

$$\mathbb{S}[\beta v(t)] = \frac{u}{\beta}V\left(\frac{s}{\beta}, u\right)$$

Using the Definition 1.1 of Shehu transform, we deduce

$$\mathbb{S}[\beta v(t)] = \int_0^\infty e^{\left(\frac{-st}{u}\right)}v(\beta t)dt \qquad 1.4$$

Substituting $x = \beta t \Rightarrow t = \frac{x}{\beta}$ and $\frac{dt}{dx} = \frac{1}{\beta} \Rightarrow dt = \frac{dx}{\beta}$ in equation 1.4 yield

$$\mathbb{S}[\beta v(t)] = \frac{1}{\beta}\int_0^\infty e^{\left(\frac{-sx}{u\beta}\right)}v(x)dx$$

$$= \frac{1}{\beta}\int_0^\infty e^{\left(\frac{-st}{u\beta}\right)}v(t)dt$$

$$= \frac{u}{\beta}\int_0^\infty e^{\left(\frac{-st}{\beta}\right)}v(ut)dt$$

$$= \frac{u}{\beta}V\left(\frac{s}{\beta}, u\right)$$

Property3: Derivative of Shehu transform. If the function $v^{(n)}(t)$ is the nth derivative of the function $v(t) \in A$ with respect to t, then its Shehu transform is defined by

$$\mathbb{S}\left[v^{(n)}(t)\right] = \frac{s^n}{u^n}V(s,u) - \sum_{k=0}^{n-1}\left(\frac{s}{u}\right)^{n-(k+1)} v^{(k)}(0) \qquad 1.5$$

When $n = 1$, we obtain the following derivatives with respect to t.

$$\mathbb{S}\left[v^{(1)}(t)\right] = \mathbb{S}[v'(t)] = \frac{s}{u}V(s,u) - v(0) \qquad 1.6$$

When $n = 1$, we obtain the following derivatives with respect to t.

$$\mathbb{S}\left[v^{(2)}(t)\right] = \mathbb{S}[v''(t)] = \frac{s^2}{u^2}V(s,u) - \frac{s}{u}v(0) - v'(0) \qquad 1.7$$

Assume that equation 1.5 true for $n = k$. Now we want to show that for $n = k + 1$

$$\mathbb{S}\left[v^{(k+1)}(t)\right] = \mathbb{S}[(v^{(k)}(t))'] = \frac{s}{u}S\left[v^{(k)}(t)\right] - v^{(k)}(0) \text{ using equation 1.6}$$

$$= \frac{s}{u}\left[\frac{s^k}{u^k}\mathbb{S}[v(t)] - \sum_{i=0}^{k-1}\left(\frac{s}{u}\right)^{k-(i+1)} v^{(i)}(0)\right] v^{(k)}(0)$$

DOI: https://doi.org/10.29121/gjesr.v7.i8.2020.2 Impact Factor- 5.070

$$= \left(\frac{s}{u}\right)^{k+1} \mathbb{S}[v(t)] - \sum_{i=0}^{k} \left(\frac{s}{u}\right)^{k-i} v^{(i)}(0)$$

which implies that Eq (1.5) holds for
$n = k + 1$. By induction hypothesis the proof is complete

Property 4: Let the function $v(t) = 1$ be in set A. Then its Shehu transform is given by

$$\mathbb{S}[1] = \frac{u}{s}$$

Poof: Using equation 1.1

$$\mathbb{S}[1] = \int_0^\infty e^{\left(\frac{-st}{u}\right)} dt$$

$$= -\frac{u}{s} \lim_{\eta\to\infty} \left[e^{\left(\frac{-s\eta}{u}\right)}\right]_0^\infty = \frac{u}{s}$$

Property 5: Let the function $v(t) = sin(\alpha t)$ be in set A. Then its Shehu transform is given by

$$\mathbb{S}[\, sin(\alpha t)] = \frac{\alpha u^2}{s^2 + \alpha^2 u^2}$$

Property 6: Let the function $v(t) = texp(\alpha t)$ and $v(t) = texp(\alpha t)$ be in set A. Then its Shehu transform is given by $\frac{u^2}{(s-\alpha u)^2}$ and $\frac{u}{(s-\alpha u}$ respectively

Property 7: Let the function $v(t) = t^n$ be in set A. Then its Shehu transform is given by

$$\mathbb{S}[t^n\,] = n! \left(\frac{u}{s}\right)^{n+1} \qquad \text{for } n = 0, 1, 2 \ldots$$

Shehu transform for handling system of linear ordinary differential equations

Example 1: Now consider the following system of differential equations

$$\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = x - y \end{cases} \qquad 1.8 \text{ with initial condition } x(0) = 1, y(0) = 2$$

Solution: Taking the Shehu transform equation (1.8) and using the property of Shehu transform we have:

$$\frac{s}{u} X(s,u) - x(0) = X(s,u) \qquad 1.8\,*$$

$$\frac{s}{u} Y(s,u) - y(0) = X(s,u) - Y(s,u) \qquad 1.8\,**$$

Using the initial condition $x(0) = 1$ from (1.8 $*$)and $y(0) = 2$ from 1.8 $**$ we obtained:

$$\frac{s}{u} X(s,u) - 1 = X(s,u) \qquad 1.9\,*$$

$$\frac{s}{u} Y(s,u) - 2 = X(s,u) - Y(s,u) \qquad 1.9\,**$$

From 1.9 $*$ we get, $X(s,u) = \frac{u}{s-u}$

and therefore, by the inverse Shehu transform, we obtain that

$$x(t) = e^x$$

Now Substituting $X(s,u) = \frac{u}{s-u}$, we obtained:

$$Y(s,u) = \frac{u(2s-u)}{(s-u)(s+u)} = \frac{u}{2(s-u)} + \frac{3u}{2(s+u)}$$

and therefore, by the inverse Shehu transform, we obtain that

$$y(t) = \frac{1}{2} e^x + \frac{3}{2} e^{-x}$$

DOI: https://doi.org/10.29121/gjesr.v7.i8.2020.2 Impact Factor- 5.070

Therefore, system (1.8) has the following pair of solutions:

$$x(t) = e^{x} \;\; y(t) = \frac{1}{2}e^{x} + \frac{3}{2}e^{-x}$$

Example 2: Solve the following system of differential equations

$$\begin{cases} \dfrac{dx}{dt} + x = y(t) + e^{t} \\ \dfrac{dy}{dt} + y = x + e^{t} \end{cases} \qquad 1.10$$

With initial conditions $y(0) = x(0) = 1$

We apply the Shehu transform to the DEs and for simplicity we write X and Y instead of $X(s,u)$ and $Y(s,u)$

$$\begin{cases} \dfrac{s}{u}X - x(0) + X = Y + \dfrac{u}{s-u} \\ \dfrac{s}{u}Y - y(0) + Y = X + \dfrac{u}{s-u} \end{cases} \qquad 1.11$$

Using the given initial condition equation (1.11) is equal to

$$\frac{s}{u}X - 1 + X = Y + \frac{u}{s-u} \qquad 1.11*$$

$$\frac{s}{u}Y - 1 + Y = X + \frac{u}{s-u} \qquad 1.11**$$

Now, by rearranging 1.11 ∗ we do have:

$$X\left(\frac{s}{u} + 1\right) = Y + \frac{u}{s-u} + 1$$

$$\Rightarrow X = \left(\frac{u}{s+u}\right)Y + \frac{u^2}{(s-u)(s+u)} + \frac{u}{s+u}$$

Now, the value of $X = \left(\frac{u}{s+u}\right)Y + \frac{u^2}{(s-u)(s+u)} + \frac{u}{s+u}$ from 1.11 ∗∗

$$\Rightarrow \frac{s}{u}Y - 1 + Y = \left(\frac{u}{s+u}\right)Y + \frac{u^2}{(s-u)(s+u)} + \frac{u}{s+u} + \frac{u}{s-u}$$

$$\Rightarrow \left(\frac{s}{u} + 1 - \frac{u}{s+u}\right)Y = \frac{u^2}{(s-u)(s+u)} + \frac{u}{s+u} + \frac{u}{s-u} + 1$$

After, some simplification we get

$$\Rightarrow Y\left(\frac{s^2+2su}{u(s+u)}\right) = \frac{s^2+2su}{(s-u)(s+u)}$$

$$\Rightarrow Y = \frac{u}{s-u}$$

and therefore, by the inverse Shehu transform, we obtain that

$$y(t) = e^{t}$$

Now, substituting the value of $Y = \frac{u}{s-u}$ in to $X = \left(\frac{u}{s+u}\right)Y + \frac{u^2}{(s-u)(s+u)} + \frac{u}{s+u}$, we have

$$X = \left(\frac{u}{s+u}\right)\left(\frac{u}{s-u}\right) + \frac{u^2}{(s-u)(s+u)} + \frac{u}{s+u}$$

$$\Rightarrow X = \frac{2u^2}{(s-u)(s+u)} + \frac{u}{s+u}$$

$$\Rightarrow X = \frac{2u^2 + u(s-u)}{(s-u)(s+u)} = \frac{u}{s-u}$$

So, by the inverse Shehu transform we get that

$$x(t) = e^{t}$$

[Andualem, 7(8): August 2020] ISSN 2348 – 8034
DOI: https://doi.org/10.29121/gjesr.v7.i8.2020.2 Impact Factor- 5.070

Therefore, system (1.10) has the following pair of solutions

$$y(t) = e^t$$
$$x(t) = e^t$$

III. CONCLUSION

In this paper, we have successfully discussed the Shehu transform for the solution of system of differential equations. We also successfully found an exact solution in all examples.

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